# UPPSC-AE 2025

## Uttar Pradesh
## Public Service Commission

Combined State Engineering Services Examination

**Assistant Engineer**

# Mechanical Engineering

## Mechanics of Solids

Well Illustrated **Theory** *with*
**Solved Examples** and **Practice Questions**

MADE EASY PUBLICATIONS

# Mechanics of Solids

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CHAPTER 1

# Properties of Metals, Simple Stress-Strain and Elastic Constants

## 1.1 Introduction

Strength of material is a branch of applied mechanics that deals with the behaviour of solid bodies subjected to various types of loading and internal forces developed due to these loading. The objective of our analysis is to determine the stresses, strains and deflections produced by the loads in different structures and loading conditions.

## 1.2 Normal Stress

- It is the internal resistance offered by the body against external loading or deformation which is force per unit area.
- Stress induced in the material depends upon the nature of force, point of application and cross-section area of material. Stress can be tensile or compressive in nature depending on the nature of load.

$$\sigma = \frac{P}{A} \text{ N/mm}^2 \text{ or MPa}$$

- Sign convention: Tensile stress = +ve; Compressive stresses = –ve
- Stresses are induced only when motion of bar is restricted either by some force or reaction induced. If a body or bar is free to move or free expansions allowed then no stresses will be induced.

**NOTE:** Pressure has same unit but pressure is different physical quantity than stress. Pressure is external normal force distributed over surface.

On the basis of cross-sectional area considered during calculation of stresses, direct stresses can be of following two types:

1. Engineering stress or nominal stress
2. True stress or Actual stress

- **Engineering stress (Nominal stress)**

Mathematically, $\sigma = \frac{P}{A_0}$ where, $A_0$ = original cross-sectional area of specimen taken

- **True stress (Actual stress)**

Mathematically, $\sigma = \frac{P}{A_a}$ where, $A_a$ = Actual cross-sectional area of specimen at any time of loading i.e. changed area of cross-section due to loading

$$A_a = A_0 \pm \Delta A$$ '+' for Compression, '–' for Tension

**Remember**

- In tension, true or actual stress is always greater than engineering or nominal stress.
- In compression, true or actual stress is always less than engineering or nominal stress.

## 1.3 Strain

Strain is a measure of deformation representing the displacement between particles in the body relative to a reference length.

Mathematically strain can be calculated as

$$\in = \frac{\Delta L}{L}$$

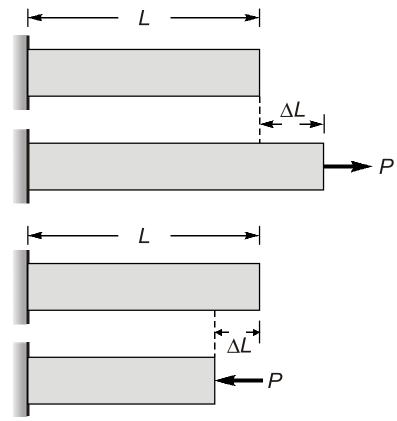



Fig.

**Unit:** Strain is dimensionless quantity. It is always expressed in the form of number. If the member is in tension, the strain is called a tensile strain. If the member is in compression, the strain is called a compressive strain.

On the basis of length of member used in calculation of strain, strain can be of following two types:

1. Engineering or nominal strain
2. True or Actual strain

**Engineering or Nominal Strain**

Engineering or nominal strain is strain calculated, when length of member is taken as original length

$$\in_0 = \frac{\Delta l}{l_0} \qquad l_0 = \text{original length of member}$$

**True or Actual Strain**

True or actual strain is strain calculated, when length of member is taken as actual length of member at loading

$$\in_a = \frac{\Delta l}{l_a} \qquad l_a = \text{Actual length of member}$$

$$l_a = l_0 \pm \Delta l$$

Sign convention:

- Tensile strains are positive whereas compressive strains are negative.

## 1.4 Stress-Strain Curve

1. **Elastic Behavior:** Elastic behavior of the material occurs when the strains in the specimen are within the I[st] region shown in Fig. Here the curve is actually a *straight line* throughout most of this region, so that the stress is *proportional* to the strain. The material in this region is said to be *linear elastic*. The upper stress limit to this linear relationship is called the ***proportional limit***, $\sigma_{pl}$. If the stress slightly exceeds the proportional limit, the curve tends to bend and flatten out as shown. This continues until the stress reaches the ***elastic limit***.

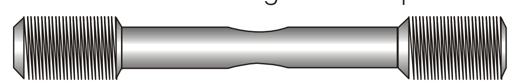
   Upon reaching this point, if the load is removed the specimen will still return back to its original shape. Normally for steel, however, the elastic limit is seldom determined, since it is very close to the proportional limit and therefore rather difficult to detect.

2. **Yielding:** A slight increase in stress above the elastic limit will result in a breakdown of the material and cause it to *deform permanently*. This behavior is called ***yielding***, and it is indicated by the

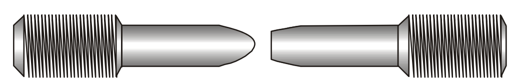
rectangular II$^{nd}$ region of the curve. The stress that causes yielding is called the ***yield stress*** or ***yield point***, $\sigma_y$ and the deformation that occurs is called ***plastic deformation***. Although not shown in Fig. for low carbon steels or those that are hot rolled, the yield point is often distinguished by two values. The ***upper yield point*** occurs first, followed by a sudden decrease in load-carrying capacity to a ***lower yield point***. Notice that once the yield point is reached, then as shown in Fig. the specimen will continue to elongate (strain) *without any increase in load*. When the material is in this state, it is often referred to as being ***perfectly plastic***.

3. Strain Hardening: When yielding has ended, an increase in load can be supported by the specimen, resulting in a curve that rises continuously but becomes flatter until it reaches a maximum stress referred to as the ***ultimate stress***, $\sigma_u$. The rise in the curve in this manner is called ***strain hardening***, and it is identified in Fig. as the III$^{rd}$ region.

4. Necking: Up to the ultimate stress, as the specimen elongates, its cross-sectional area will decrease. This decrease is fairly uniform over the specimen's entire gauge length; however, just after, at the ultimate stress, the cross-sectional area will begin to decrease in a localized region of the specimen. As a result, a constriction or "neck" tends to form in this region as the specimen elongates further as shown in, Fig. (a). This region of the curve due to necking is indicated as IV$^{th}$ region in Fig. Here the stress–strain diagram tends to curve downward until the specimen breaks at the ***fracture stress***, $\sigma_f$ as shown in Fig. (b).

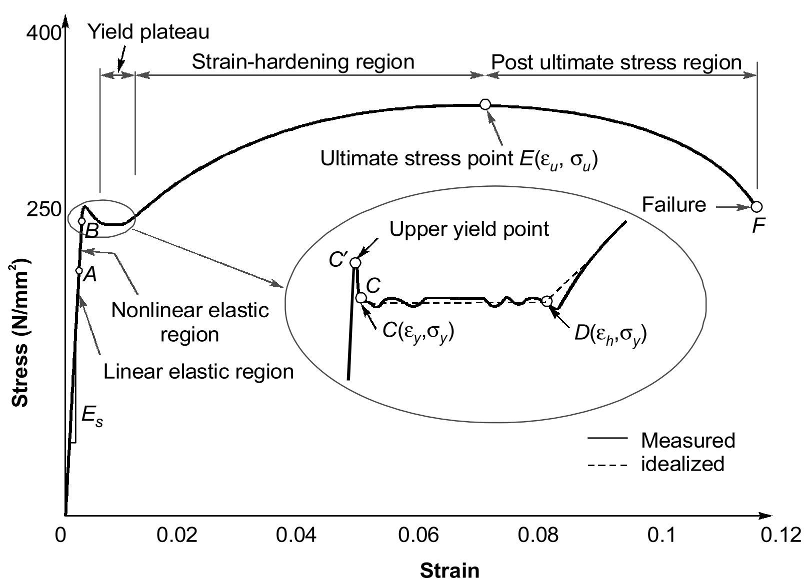


(a) Necking

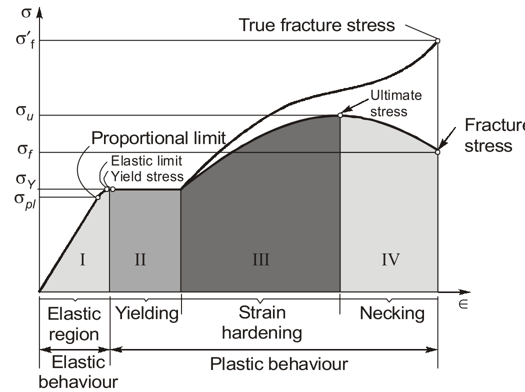


(b) Failure of a ductile material (formation of a cup-cone shape at the fracture location)

**Fig.**




**True Stress–Strain Diagram:** Instead of always using the original cross-sectional area and specimen length to calculate the (engineering) stress and strain, we could have used the actual cross-sectional area and specimen length at the instant the load is measured. The values of stress and strain found from these measurements are called true stress and true strain, and a plot of their values is called the ***true stress–strain diagram***. Note that the conventional and true $\sigma$–$\in$ diagrams are practically coincident when the strain is small. The differences between the diagrams begin to appear in the strain-hardening range, where the magnitude of strain becomes more significant. In particular, there is a large divergence within the necking region. Here it can be seen from the conventional σ–∈ diagram that the specimen actually supports a *decreasing load*, since $A_0$ is constant when calculating engineering stress, $\sigma = P/A_0$. However, from the true σ–∈ diagram, the actual area *A* within the necking region is always decreasing until fracture, $\sigma'_f$ and so the material actually sustains *increasing stress*, since $\sigma = P/A$.



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

**DO YOU KNOW?**

Strain that occurs before the yield point is called elastic strain and that which occurs after yield point with no increase in stress is called plastic strain. For mild steel, plastic strain is 10 to 15 times of elastic strain.

- Ideal curve for tension is shown in the figure. However, actual behaviour is different and indicates apparently reduced yield stress in compression for mild steel. The divergence between tension and compression results is explained by Bauchinger and is called **Bauchinger effect**.

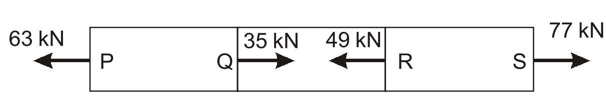
**Example - 1.1** A rod of dimension 20 mm × 20 mm is carrying an axial tensile load of 10 kN. The tensile stress developed is

(a) 0.025 MPa (b) 0.25 MPa
(c) 25 MPa (d) 250 MPa

**Solution: (c)**

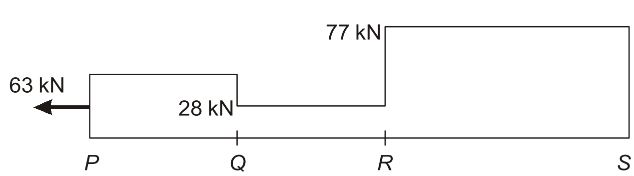
$$\text{Induced stress, } \sigma = \frac{P}{A} = \frac{10 \times 1000}{20 \times 20} = 25 \text{ MPa}$$

**Example - 1.2** A cross-section bar of area 700 mm$^2$ is subjected to an axial load as shown in the figure below, what is the value of stress (MPa) in the section QR?




(a) 30 (b) 40
(c) 50 (d) 60

**Solution: (b)**



***Axial loading diagram***

load in *QR* section, $P_{QR} = +28$ kN

$$\text{Induced stress, } \sigma_{QR} = \frac{PQR}{AQR} = \frac{28 \times 10^3}{700} - 40 \text{ MPa}$$

### 1.4.1 Stress Strain Curve for Other Grades of Steel in Tension

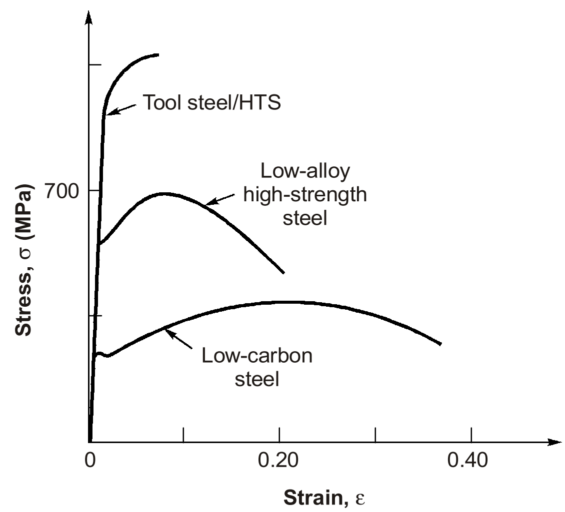



**Fig.** *(a) Tensile stress-strain diagram for different grades of steel*

**Remember**

- All the grade of steel have same Young's modulus of elasticity.
- Among all grades high tension steel (HTS) is more brittle and mild steel is more ductile.
- High tension steel has higher ultimate strength than other grade of steel.

### 1.4.2 Stress Strain Curve for Various Materials

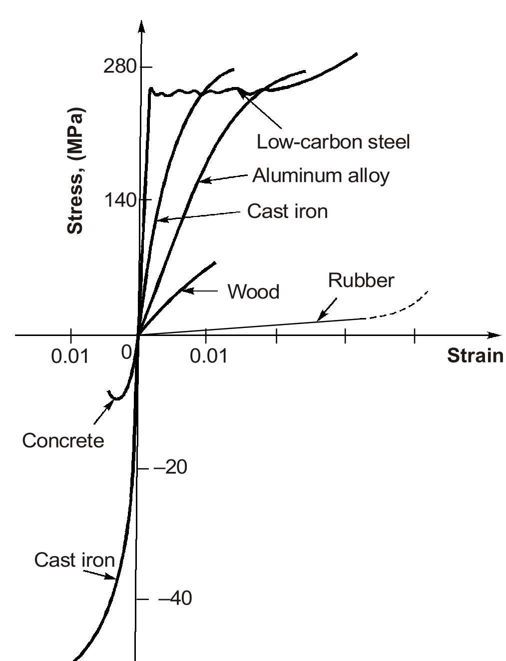



**Fig.** *(b) Stress-strain diagram for different material*

## 1.5 Properties of Metals

### 1.5.1 Ductility

- Ductility is the property by which material can be stretched.
- Large deformations are thus possible in ductile materials before the absolute failure or rupture takes place.
- Some of the examples are mild steel, aluminium, copper, manganese, lead, nickel, brass, bronze, monal metal etc.

**Example - 1.3** The percentage reduction in area in case of cast iron when it is subjected to tensile test is of the order of

(a) 0% (b) 10%
(c) 20% (d) 25%

**Solution: (a)**
The % reduction in area in case of cast iron when it is subjected to tensile test is of the order of 0%, because cast iron being brittle material does not undergo elongation.

### 1.5.2 Brittleness

- Brittleness is the lack of ductility i.e. material cannot be stretched.
- In brittle materials, failure takes place with a relatively smaller deformation. This property is undesirable.
- For brittle materials fracture point and ultimate points are same, and after proportional limit very small strain is seen. Some of the examples are cast iron, concrete and glass.

**NOTE**

- To distinguish between these two types of materials, materials with strain less than 5% at fracture point are regarded as brittle and those having strains greater than 5% at fracture point are called ductile (this value for mild steel at fracture is about 25%).

### 1.5.3 Malleability

- It is the property by which a material can be uniformly extended in a direction without rupture.
- A malleable material possesses a high degree of plasticity. This property is of great use in operations like forging, hot rolling, drop (stamping) etc.

### 1.5.4 Toughness

- The property which enables material to absorb energy without fracture. This property is very desirable in case of cyclic loading or shock loading.
- The **modulus of toughness** is measured as area under entire stress-strain curve and it is the energy absorbed by material of the specimen per unit volume upto fracture point.

$$\text{Modulus of toughness} = \left[\frac{\sigma_y + \sigma_u}{2}\right] \epsilon_f$$

Where, $\sigma_y$ = Yield tensile strength; $\sigma_u$ = Ultimate tensile strength; $\epsilon_f$ = Strain at fracture point

- The modulus of toughness will depend upon ultimate tensile strength and strain at failure (fracture strain). Hence the material which is very ductile will exhibit a higher modulus of toughness as the case with mild steel.
- The **modulus of resilience** is the maximum elastic energy per unit volume that can be absorbed without attaining plastic stage.

$$\text{Modulus of resilience } (u) = \frac{\sigma_y^2}{2E}$$

- **The modulus of resilience** depends upon yield strength and hence a material with higher yield strength will have higher modulus of resilience.

**NOTE:** Higher toughness is a desirable property in materials used for gears, chains, crane hooks, freight car etc. Higher resilience is desirable in springs.

### 1.5.5 Hardness

- Hardness is defined as the resistance to indentation or scratching or surface abrasion.
- There are two methods of hardness measurement:
  1. Scratch hardness - commonly measured by Mohs test.
  2. Indentation hardness (abrasion) measured by
     (a) Brinell hardness method (b) Rockwell hardness
     (c) Vickers hardness (d) Knoop hardness

**NOTE:** It should be noted that ductile materials are tough and brittle materials are hard.

### 1.5.6 Fatigue

- The behaviour of material under variable loading (dynamic loading) is referred to as **fatigue**.
- Factors affecting fatigue are:
  1. Loading conditions 2. Frequency of loading 3. Corrosion
  4. Temperature 5. Stress concentration

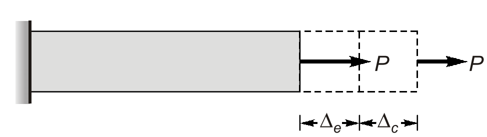
### 1.5.5 Creep

where,

$\Delta_e$ = Elastic deflection = $\frac{PL}{AE}$

$P$ = Static load

$\Delta_c$ = Deformation due to creep



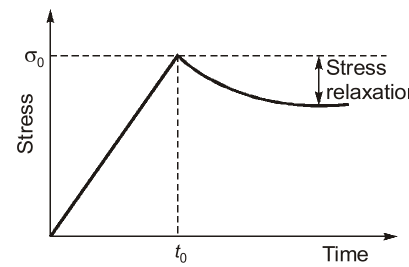

Creep is permanent deformation which is recorded with passage of time at constant loading. Total creep deformation continues to increase with time asymptotically.

Factors affecting creep are as follows:

1. Magnitude of load
2. Type of loading (static or dynamic)
3. Time or Age of loading
4. Temperature
   - At higher temperature, due to greater mobility of atoms most of the materials loose their strength and elastic constants also get reduced. Hence, greater deformation at elevated temperature results, even under constant loading. Therefore, creep is more pronounced at higher temperature, and thus it must be considered for design of engines and furnaces.
   - Temperature at which the creep becomes very appreciable is half of the melting point temperature on absolute scale and known as ***homologous temperature***.

### 1.5.6 Stress Relaxation

If a wire of metal is stretched between two immovable supports, so that it has an initial tension $\sigma_0$. The stress in the wire gradually diminishes, eventually reaching a constant value. This process, which is manifestation of creep is called **stress relaxation**. (This is the reason why electric wires sag after long time).




## 1.6 Hook's Law

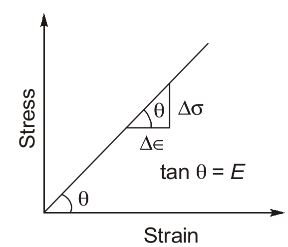
- It states that within elastic limit, stress for most of the metals is proportional to strain upto proportion at limit.

$$\text{Stress} \propto \text{strain}$$

$$\sigma = E\epsilon$$

$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant} = E$$

Where, $E$ = Young's modulus of elasticity




- Hook's law is valid upto limit of proportionality. However for mild steel proportional limit and elastic limit are almost equal. But for other metals and materials elastic limit may be higher than proportional limit. e.g. rubber.
- The slope of stress-strain curve is called modulus of elasticity ($E$). The **modulus of elasticity** ($E$) is the constant of proportionality which is defined as the intensity of stress that causes unit strain. Thus, modulus of elasticity ($E$) has the units same as units of stress.

**Assumptions in Hooke's Law:**

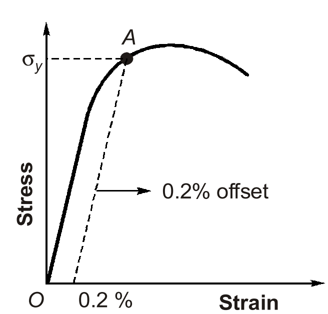
1. Material is homogeneous (properties are equal at all points).
2. Material is isotropic (properties are equal in all direction).
3. Material is elastic.

**NOTE**

1. If elastic properties are different in three mutually perpendicular direction then material is called **Orthotropic material**, e.g. plywood.
2. If properties are different in all direction then material is called Non-isotropic or Anisotropic material, e.g., Crystal.

## 1.7 Proof Stress

- When a material such as **aluminium** does not have an obvious yield point and yet undergoes large strains after the proportional limit is exceeded, an arbitrary yield stress may be determined by the **offset method**.
- A line parallel to initial linear part is drawn, which is offset by some standard amount of strain such as 0.2%. The intersection of the offset point ($B$) defines the yield stress or offset yield stress, which is slightly above the proportional limit and is called **proof stress**.



**Example - 1.10** The strain energy per unit volume of a round bar under uni-axial tension with axial stress σ and modulus of elasticity *E* is

(a) $\frac{\sigma^2}{E}$ (b) $\frac{\sigma^2}{2E}$

(c) $\frac{\sigma^2}{3E}$ (d) $\frac{\sigma^2}{4E}$

**Solution: (b)**

$$\text{Strain energy/volume} = \frac{1}{2}\times\sigma\times E = \frac{1}{2}\times\sigma\times\frac{\sigma}{E} = \frac{\sigma^2}{2E}$$

**Example - 1.11** Total area under the stress-strain curve of a mild steel specimen tested up to failure under tension is a measure of its

(a) Breaking strength (b) Toughness

(c) Hardness (d) Stiffness

**Solution: (b)**

## Student's Assignment

**Q.1** The ratio of Young's modulus to modulus of rigidity for a material having Poisson's ratio 0.2 is

(a) $\frac{12}{5}$ (b) $\frac{5}{12}$

(c) $\frac{5}{14}$ (d) $\frac{14}{5}$

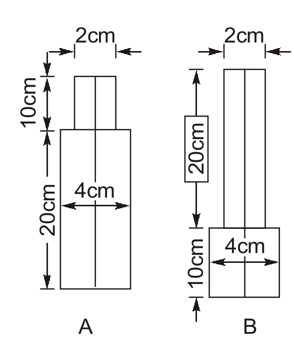
**Q.2** A straight wire 15 m long is subjected to tensile stress of 2000 kgf/cm$^2$. Elastic modulus is $1.5\times10^6$ kgf/cm$^2$. Coefficient of linear expansion for the material is $16.66\times10^{-6}$/°F. The temperature change (in °F) to produce the same elongation as due to the 2000 kgf/cm$^2$ tensile stress in the material is:

(a) 40 (b) 80

(c) 120 (d) 160

**Q.3** Two similar round bars *A* and *B* are each 30 cm long as shown in the given figure. The ratio of strain energies stored by the bars *A* and *B*, $\frac{U_B}{U_A}$ is

(Assume both bars to be axially loaded equally)




(a) 3/2 (b) 1.0

(c) 5/8 (d) 2/3

**Q.32** Which one of the following pairs is NOT correctly matched?
(a) Visco-elastic – small plastic zone
(b) Orthotropic material – different properties in three perpendicular directions
(c) Strain hardening material – stiffening effect felt at some stage
(d) Isotropic material – same physical property in all directions at a point

**Q.33** The effect of a force on a body depends on its:
(a) direction (b) magnitude
(c) position (d) all of these

**Q.34** A solid metal bat of uniform diameter $D$ and length $L$ is hung vertically from a ceiling. If the density of the material of the bar is $\gamma$ and the modulus of elasticity is $E$, then the total elongation of the bar due to its own weight will be ____________.
(a) $\gamma L/2E$ (b) $\gamma L^2/2E$
(c) $\gamma E/2L$ (d) $\gamma E/2L^2$

**Q.35** A bar of diameter 30 mm is subjected to a tensile load such that the measured extension on a gauge length of 200 mm is 0.09 mm and the change in diameter is 0.0045 mm. What will the Poissons ratio?
(a) 1/3 (b) 1/4
(c) 1/5 (d) 1/6

**Q.36** An experiment was done and it was found that the bulk modulus of a material is equal to its shear modulus. Then what will be its Poisson ratio?
(a) 0.125 (b) 0.150
(c) 0.200 (d) 0.375

**Q.37** What will be the approximate value of shear modulus of a material if the modulus of elasticity is 200 GN/m$^2$ and its Poissons ratio is 0.30?
(a) 76.9 GN/m$^2$ (b) 80 GN/m$^2$
(c) 93.3 GN/m$^2$ (d) 103.9 GN/m$^2$

**Q.38** What is the bulk modulus of elasticity?
(a) The ratio of shear stress to shear strain
(b) The ratio of direct stress to direct strain
(c) The ratio of volumetric stress to volumetric strain
(d) The ratio of direct stress to volumetric strain

**Q.39** E, G, K and $\mu$ elastic modulus, shear modulus, bulk modulus and Poisson's ratio respectively. To express the stress strain relations completely for this material
(a) E, G and $\mu$ must be known
(b) E, K and $\mu$ must be known
(c) Any two of the four must be known
(d) All the four must be known

**Q.40** The ability of a material to undergo plastic deformation without rupture, when a compressive force is applied, is known as ________.
(a) Ductility (b) Formability
(c) Brittleness (d) Malleability **[UPPSC]**

**Q.41** With an increase in temperature, malleability of material _________.
(a) Increases
(b) Decreases
(c) Remains constant
(d) Independent of temperature

**Q.42** Stress-strain analysis is done to know which of the following properties of materials?
(a) Physical properties
(b) Optical properties
(c) Mechanical properties
(d) Magnetic properties

**Q.43** The failure criterion for ductile materials is based on
(a) yield strength (b) ultimate strength
(c) shear strength (d) limit of proportionality

■■■■

## ANSWER KEY // STUDENT'S ASSIGNMENT

| | | | | |
|---|---|---|---|---|
| **1.** (a) | **2.** (b) | **3.** (a) | **4.** (a) | **5.** (a) |
| **6.** (c) | **7.** (c) | **8.** (b) | **9.** (c) | **10.** (b) |
| **11.** (b) | **12.** (d) | **13.** (c) | **14.** (d) | **15.** (b) |
| **16.** (c) | **17.** (a) | **18.** (b) | **19.** (d) | **20.** (a) |
| **21.** (c) | **22.** (d) | **23.** (b) | **24.** (c) | **25.** (a) |
| **26.** (a) | **27.** (a) | **28.** (b) | **29.** (b) | **30.** (d) |
| **31.** (c) | **32.** (a) | **33.** (d) | **34.** (b) | **35.** (a) |
| **36.** (a) | **37.** (a) | **38.** (d) | **39.** (c) | **40.** (d) |
| **41.** (a) | **42.** (c) | **43.** (a) | | |

## HINTS & SOLUTIONS / STUDENT'S ASSIGNMENT

**1. (d)**

$$G = \frac{E}{2(1+\mu)} \Rightarrow \frac{E}{G} = 2(1+\mu)$$

$$\frac{E}{G} = 2\times 1.2 = 2.4 = \frac{12}{5}$$

**2. (b)**

$$\Delta L = \frac{\sigma L}{E} = L\alpha\Delta T$$

$$\therefore \quad \Delta T = \frac{\sigma}{E\alpha} = \frac{2000}{1.5\times 10^{6}\times 16.66\times 10^{-6}}$$

$$= 80.03^\circ \text{F}$$

**3. (a)**

$$U = \frac{P^2L}{2AE} \text{ For axially loaded bar.}$$

$$U_A = \frac{P^2L_1}{2A_1E} + \frac{P^2L_2}{2A_2E}$$

$$= \frac{P^2}{2E}\left[\frac{10}{2^2} + \frac{20}{4^2}\right]$$

$$U_B = \frac{P^2L_1}{2AE} + \frac{P^2L_2}{2A_2E}$$

$$= \frac{P^2}{2E}\left[\frac{20}{2^2} + \frac{10}{4^2}\right]$$

$$\frac{U_B}{U_A} = \frac{\left[\frac{20}{2^2} + \frac{10}{4^2}\right]}{\left[\frac{10}{2^2} + \frac{20}{4^2}\right]}$$

$$= \left(\frac{4\times 20 + 10}{4\times 10 + 20}\right)$$

$$= \frac{90}{60} = 1.5 = \frac{3}{2}$$

**4. (a)**

1-E, 2-D, 3-C, 4-B, 5-A

Toughness is measured by area under stress-strain curve. Brittleness is measured by strain at failure. Higher the yield point, the metal will be harder. Strong/weak can be measured by ultimate tensile strength or peak of stress-strain curve. Based on these concepts the answer can be easily found.

**5. (a)**

$$\Delta = \frac{p_1 l_1}{EA} = \frac{p_2 l_2}{EA}$$

$$\Rightarrow \quad \frac{p_1}{p_2} = \frac{l_2}{l_1} = \frac{6}{4} = \frac{3}{2}$$

**6. (c)**

The bar will remain straight with some inclination $\theta$. Therefore from geometry.

$$\therefore \quad \frac{\Delta_c}{1} = \frac{\Delta_s}{2}$$

$$\frac{\Delta_s}{\Delta_c} = 2$$

**7. (c)**

Net strain in lateral direction

$$\epsilon_2 = \frac{1}{E}\left[-\sigma_2 + \mu(\sigma_1+\sigma_2)\right]$$

Since all the stresses are compressive.

When $\sigma_1$ alone is acting compressive

$$\epsilon_2' = +\frac{\mu\sigma_1}{E}$$

Given, $$\epsilon_2 = \frac{\epsilon_2'}{2}$$

$$-\sigma_2 + \mu(\sigma_1+\sigma_2) = +\frac{\mu\sigma_1}{2}$$

$$2\sigma_2(1-\mu) = \mu\sigma_1$$

$$\sigma_2 = \frac{\mu}{2(1-\mu)}\sigma_1$$

**8. (b)**

Grey cast iron has very poor toughness. Its major reason is notch like an effect of graphite flakes. These flakes increase stress concentration in it lowering toughness.

**9. (c)**

Due to presence of notch in the specimen or workpiece. Ductile material fails as brittle fracture.